HW 4, Problem 4.b supplement

Some people asked why they couldn't just say their integrals in 4.b were not equal because they were "different". In my solution, I show that the difference is nonzero, thus they are not equal. Here is an example where I have two integrals similar to 4.b, but they are equal.

Let $x(t) = \int_t^\infty (\lambda-1)e^{-(\lambda-1)^2}\,d\lambda$ and $y(t) = -\int_{-\infty}^t (\lambda-1)e^{-(\lambda-1)^2}\,d\lambda$

does $x(t) = y(t)$?

$$x - y = \int_t^\infty (\lambda-1)e^{-(\lambda-1)^2}\,d\lambda - -\int_{-\infty}^t (\lambda-1)e^{-(\lambda-1)^2}\,d\lambda$$

$$= \int_t^\infty (\lambda-1)e^{-(\lambda-1)^2}\,d\lambda + \int_{-\infty}^t (\lambda-1)e^{-(\lambda+1)^2}\,d\lambda$$

$$= \int_{-\infty}^\infty (\lambda-1)e^{-(\lambda-1)^2}\,d\lambda = \int_{-\infty}^\infty \lambda e^{-\lambda^2}\,d\lambda = -\frac{1}{2}e^{-\lambda^2}\Big]_{\lambda=-\infty}^{\infty} = 0$$

Of course, in this example we can actually write down x & y without integrals.

$$x(t) = \int_t^\infty (\lambda-1)e^{-(\lambda-1)^2}\,d\lambda = \int_{t-1}^{\infty-1} \lambda e^{-\lambda^2}\,d\lambda = -\frac{1}{2}e^{-\lambda^2}\Big]_{\lambda=t-1}^{\infty} = 0 - -\frac{1}{2}e^{-(t-1)^2}$$

$$= \frac{1}{2}e^{-(t-1)^2}$$

$$y(t) = -\int_{-\infty}^t (\lambda-1)e^{-(\lambda-1)^2}\,d\lambda = -\int_{-\infty-1}^{t-1} \lambda e^{-\lambda^2}\,d\lambda = +\frac{1}{2}e^{-\lambda^2}\Big]_{\lambda=-\infty}^{t-1} = \frac{1}{2}e^{-(t-1)^2} - 0$$

$$= \frac{1}{2}e^{-(t-1)^2}$$